# Analysis of the two-channel optical Bell test, using multivectors and matrices

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## Introduction

I have had it up to my eyeballs with physicists who believe in magical “instantaneous action at a distance.”

Yes, I said *magical*.

Some will fault me for referring to what appears in “serious” scientific journals [1] as *magic*, but it is nothing less. No physicist should believe in “action at a distance,” and if they do they should be looking for another career. People like Maxwell and Heaviside went to enormous trouble to base physics on action by direct contact. Maxwell’s theory of electromagnetism is thus *explicitly* one of action by direct contact,[2] and the electromagnetic field obeys Newton’s laws of motion. Einstein’s attempt at a theory of gravity, which as far as I know is the best we have so far, is also based *explicitly* on action by direct contact, and therefore predicts gravity waves.

Yet we are told by physicists that two or more pulses of light can be “entangled,” thus forming a single physical entity spanning any distance whatsoever. When one of the “entangled” pulses is resolved into a distinct pulse, supposedly the others “*instantaneously*,” over even light years of distance, also resolve into distinct pulses. The pulses do this, even though such “action at a distance” cannot possibly be consistent with Maxwell’s theory of electromagnetism. There is no possible mathematical resolution to this puzzle, because Maxwell’s theory is *explicitly* one of action by direct contact.

No wonder it is, then, when I was taught “Engineering Physics” while Alain Aspect was publishing his famous (but in multiple ways incorrect) results, I was taught the following: that “quantum physics” made no logical sense and I should not try to understand it. In fact I was *ordered* in a loud voice not to try to understand it, and *ordered* “*Just do the calculations!*” The professor also mocked Albert Einstein for resistance to the new doctrines and for Aspect’s “proof” that “*Einstein was wrong!*” I was being taught not science but an orthodox liturgy. This is the legacy of the 1927 Solvay conference and the consensus that came forth from it.

Where logic can resolve a matter, though, a consensus is worth nothing. All that matters is the logic.

"Quantum physics" does not deserve the title "science." It is a quasi-religion and a pseudoscience. Dirac notation is a liturgical language for what actually are propositions of probability theory, but physicists transmogrify these propositions, by religious incantations written in Dirac *bras* and *kets*, into "entangled" entities.

The "quantum" pseudoscience not only costs university students and their families millions of dollars, but threatens world peace. How much does the United States Department of Defense spend on "quantum computers"? Yet "quantum computers" are junk that cannot possibly function as physicists predict. There is no such thing as "entanglement," so the registers that supposedly operate in unit time must require at least polynomial time, and a device being "quantum" does not free us of the need to collect entropy for random number generation. We should not tolerate this state of affairs, in which students are fleeced and world peace is threatened. If every "quantum physicist" should be laughed out of a job, there would be many more competent persons who could approach the same empirical problems using *real* physics and actual mathematics (rather than enchanted liturgy).

The Dirac notation is really just an obfuscated notation for probability theory. What is needed, in part, is people who can do probability theory correctly and who do not need Dirac notation to do it!

A physicist of today will sometimes brag that the Dirac notation represents a complex tensor or "Hilbert" space, as if this made the mathematics more than just probability theory. In fact, this seems to be what some believe, that the complex tensor space is not merely probability theory, but indeed a magical phase of existence in which "instantaneous action at a distance" occurs. But such a tensor space is merely a mathematical construct of convenience. It has no physical meaning. I will demonstrate this below by analyzing the two-channel optical Bell test (the Aspect experiment) using not a complex tensor space, but something even fancier. I will use a tensor space of *multivectors*.

# Multivectors and geometric algebra

## Vector products and grade

Ordinary Gibbs-Heaviside vectors are nice enough, but, like physicists, we want to impress the plebeians. Physicists do this by using complex numbers to represent rotations in a plane. So did I, as a student of electrical engineering, use complex numbers this way (but without trying to impress anyone). However, are not complex numbers more properly roots of univariate real polynomials? A more direct way to represent rotations is *geometric algebra*, from which we will draw just a miniscule portion of what it contains and can do.

In geometric algebra, you can add a scalar and a vector. At least if you are as old as I am, you are likely to have an acquaintance who will argue with you vociferously that you *cannot* add a scalar and a vector. Such an acquaintance is a person who has obtained a high school or university education, but has not *overcome* that education. It is important to obtain an education, but current educations are very bad, and so it is necessary also to *overcome* the damage done. A school education *ossifies* the brain rather than make it more plastic, and also teaches much that is simply wrong, or that is limited to a context but taught as if it were universal. So wipe the prejudice from your thoughts. We will be adding things together you may have thought you could not add.

You can also multiply vectors in ways other than dot and cross products. The *geometric product* $uv$ of vectors $u$ and $v$ is

$$uv = u \cdot v + u \wedge v$$

where $u \cdot v$ is the familiar dot product, but the *wedge product* $u \wedge v$ is a new entity called a *bivector*. A bivector is a finite planar area that is oriented either counterclockwise or clockwise. You can picture a bivector as an entity normal to which is the familiar cross product.

The sum of any scalars, vectors, and bivectors is called a *multivector*. One can actually go past bivectors to any number of dimensions, and there are other ways things can be made more complicated, but we do not need any of that. The multivectors in this essay go up in complexity no farther than bivectors (and they have the usual Euclidean metric).

Let us also introduce the *grade* operator, which returns the scalar (*grade 0*), vector (*grade 1*), or bivector (*grade 2*) part of a multivector:

$$\begin{aligned} \left\langle 5 + 2e_1 + 7e_1 \wedge e_2 \right\rangle_0 &= 5 \\ \left\langle 5 + 2e_1 + 7e_1 \wedge e_2 \right\rangle_1 &= 2e_1 \\ \left\langle 5 + 2e_1 + 7e_1 \wedge e_2 \right\rangle_2 &= 7e_1 \wedge e_2 \end{aligned}$$

Given the grade operation, the dot and wedge products of two vectors can be obtained from their geometric product:

$$\begin{aligned} u \cdot v &= \langle uv \rangle_0 \\ u \wedge v &= \langle uv \rangle_2 \end{aligned}$$

Here are some identities:

$u \cdot v \;\; = \|$

## Basis vectors

Suppose we want to have an $(x,y)$ coordinate system for geometric algebra. We can do this by defining *basis vectors* $e_1$ of magnitude 1 in the $x$-direction and $e_2$ of magnitude 1 in the $y$-direction.

The following are useful identities:

$$\begin{align*} e_1 \cdot e_2 &= e_2\cdot e_1 = 0 \\ e_1 \wedge e_1 &= e_2\wedge e_2 = 0 \\ e_1 \cdot e_1 &= e_1 e_1 = e_2 e_2 = e_2\cdot e_2 = 1 \\ e_1 \wedge e_2 &= e_1 e_2 = -e_2 e_1 = -e_2\wedge e_1 \end{align*}$$

## Rotors

What if you need a unit vector other than one of the basis vectors? Complex numbers around the clock face are obtained by rotating the number 1: the number 1 is rotated counterclockwise by an angle $\varphi$ by multiplying it by $e^{i\varphi} = \cos\varphi + i\sin\varphi$. The operation is equivalent to breaking a unit vector into components.

There is an analogous operation in geometric algebra. For this operation, one uses a *rotor*. To rotate counterclockwise by $\varphi$, the rotor and its inverse are

$$\begin{align*} R_\varphi &= \cos ( \varphi / 2 ) - e_1 e_2 \sin ( \varphi / 2 ) \\ R_\varphi^{-1} &= \cos ( \varphi / 2 ) + e_1 e_2 \sin ( \varphi / 2 ) \end{align*}$$

Any vector around the clock face can be gotten by rotating $e_1$, thus:

$$\begin{align*} e_\varphi &= R_\varphi \, e_1 \, R_\varphi^{-1} \\ &= (\cos\varphi' - e_1 e_2 \sin\varphi') \, e_1 \, (\cos\varphi' + e_1 e_2 \sin\varphi') \\ &= e_1 \cos^2\varphi' - e_1 e_2 e_1 \cos\varphi'\sin\varphi' \\ &\mkern3em + e_1 e_1 e_2 \cos\varphi'\sin\varphi' - e_1 e_2 e_1 e_1 e_2 \sin^2\varphi' \end{align*}$$

where $\varphi' = \varphi/2$. Identities mentioned above allow simplifications:

$$\begin{align*} -e_1 e_2 e_1 &= e_2 \\ e_1 e_1 e_2 &= e_2 \\ - e_1 e_2 e_1 e_1 e_2 &= -e_1 \end{align*}$$

Therefore,

$$\begin{equation} e_\varphi = (\cos^2\varphi' - \sin^2\varphi')\,e_1 + (2\,\cos\varphi'\sin\varphi^{d\prime})\,e_2 \end{equation}$$

Let us refer to the *CRC Handbook of Mathematical Sciences, 6th Edition*,[3] where on page 170 we find the following double-angle relations:

$$\begin{align*} \sin 2 \mkern1mu\alpha &= 2 \mkern1mu \sin\alpha \mkern1mu\cos\alpha \\ \cos 2 \mkern1mu\alpha &= \cos^2\alpha - \sin^2\alpha \end{align*}$$

Therefore,

$$\begin{align*} e_\varphi &= e_1\cos\varphi + e_2\sin\varphi \end{align*}$$

or, in other words, we can rotate by doing component addition, just as with complex numbers, except now we are using actual vectors instead of roots of polynomials. Also we have impressed the plebeians with the fancy name “rotor” and by citing from a clinically titled reference book.

# Planes of polarization, vectors, and tensor spaces

## Notations

The “complex tensor space” of quantum mechanics may seem like magic, for no reason other than its esoteric name. Many do indeed seem to believe the space is magical, and that each point in it represents a magical being in a phase of existence where “instantaneous action at a distance” occurs. Physicists use funny *bra* and *ket* notations that are not used by mathematicians, and which therefore can be read as preternatural *physical* entities rather than as the purely *verbal* entities mathematical notations actually are.

We instead will use notations similar to those used by mathematicians. My education was in electrical engineering, with some applied mathematics (mostly numerical analysis), and my trade was computer programming, so I do not write things as “pure” mathematicians do, and cannot read their abstruse papers. But also, despite electrical engineering being applied physics, I do not like some of the liberties “pure” physicists take. With the Dirac notation, physicists have drifted so far from mathematical notations that they do not read the notations as mathematics, but instead as liturgical incantations about physical beings of a mysterious and magical nature. And they do not *reason logically* as they work, but instead *just do the calculations*.

Instead of *bras*, *kets*, and whatnot to represent tensors, all the mathematics below will employ such notations as multivectors and matrices. Thus a pulse of plane-polarized light whose plane of polarization is oriented along the $x$-axis can be represented by either $e_1$ or $-e_1$. Let us choose $e_1$. Similarly, for a pulse of light plane-polarized orthogonally to the first, the representation will be $e_2$. Now suppose “There is a light source that transmits a light pulse $e_1$ to the left arm of the apparatus and $e_2$ to the right arm.” This initial *proposition* can be written as a matrix thus:

$$\begin{equation} \text{prop}_1 = \begin{pmatrix}e_1\\e_2\end{pmatrix} \end{equation}$$

The upper row of such a matrix represents the left arm of the apparatus, and the lower row represents the right arm. The expression $\text{prop}_1$ defines a point in a tensor space. Its meaning is *not* a magical “entangled pair” of light pulses.[4] Its meaning is *a proposition*.

That a point in a space might represent something *verbal* rather than physical may seem strange to many physicists, and difficult to handle. If so, that is their problem, not ours. Let them get other jobs. They are costing university students a fortune for lessons in nonsense, and they are ruining the electronics profession and threatening global peace, with “quantum electronics” and “quantum computers” that cannot possibly work as advertised. A “quantum computer” is an expensive and unreliable `WAIT A WHILE THEN PRESS ENTER` device. The industry in such worthless machines should be shut down.

Later we will encounter matrices with multiple columns, representing propositions in tensor spaces of higher dimensionality, and weighted sums of matrices, representing propositions of probability theory. At no time will magic occur.

## Polarizing beam splitters (PBS)

The effect of a polarizing filter on plane-polarized light is described by the *Law of Malus*: the intensity of the retransmitted light is altered by a factor $\cos^2(\vartheta-\varphi)$, where $\vartheta$ is the angle of the filter’s axis of transmission and $\varphi$ is the angle of polarization of the impinging light. Let us suppose the mechanism of a polarizing filter is that a discrete light pulse either is or is not retransmitted, but retains its intensity. In that case the Law of Malus gives an *average* intensity, which can be regarded as the probability of retransmission of a light pulse.

Now let us proceed from simple polarizing filters to *polarizing beam splitters (PBS)*, also known as *beam-splitting polarizers*. These are devices made, one way or another, from two complementary polarizing filters, which split impinging light into orthogonal polarizations. In one output channel the light will be polarized along the main axis of transmission, and in the other channel along the orthogonal axis. Ideally, the average intensity of light in the orthogonal channel is altered by the factor $\sin^2 ( \vartheta-\varphi )$ , so the total average intensity is not lost. Similarly, for our probabilistic interpretation, we will make an idealistic assumption: that the light pulse is retransmitted in either the main or the orthogonal channel, but never lost. The probability of retransmission is therefore $\cos^2 ( \vartheta - \varphi ) + \sin^2 ( \vartheta - \varphi ) = 1$ .

Sometimes I visualize the action of a polarizing filter as the projection of the unit vector of the impinging light onto the axis of polarization of the filter. This gives a vector of magnitude reduced from unity, and the square of that magnitude is the (possibly probabilistic) intensity of the retransmitted light. If $e_\vartheta$ is a unit vector representing the axis of transmission of the filter, and $e_\varphi$ is the vector of polarization of the impinging light, then the projection is

$$\begin{equation} \text{proj}_{e_\vartheta}e_\varphi = (e_\varphi\cdot e_\vartheta)e_\vartheta \end{equation}$$

The probability of retransmission is, conveniently, the geometric product of this projection with itself:

$$\begin{equation} (\text{proj}_{e_\vartheta}e_\varphi)^2 = (e_\varphi\cdot e_\vartheta)^2 e_\vartheta e_\vartheta = \cos^2(\vartheta - \varphi) \end{equation}$$

The initial light pulses are plane-polarized $e_1$ and $e_2$, and in a PBS there are two polarizers: the main channel $e_\vartheta$ and the orthogonal channel $e_{\vartheta+\pi/2}$. Therefore there are the following special instances:

$$\begin{align*} (\text{proj}_{e_\vartheta}e_1)^2 &= (e_1\cdot e_\vartheta)^2 e_\vartheta^2 = \cos^2\vartheta \\ (\text{proj}_{e_\vartheta}e_2)^2 &= (e_2\cdot e_\vartheta)^2 e_\vartheta^2 = \sin^2\vartheta \\ (\text{proj}_{e_{\vartheta+\pi/2}}e_1)^2 &= (e_1\cdot e_{\vartheta+\pi/2})^2 e_{\vartheta+\pi/2}^2 = \cos^2(\vartheta+\pi/2) = \sin^2\vartheta \\ (\text{proj}_{e_{\vartheta+\pi/2}}e_2)^2 &= (e_2\cdot e_{\vartheta+\pi/2})^2 e_{\vartheta+\pi/2}^2 = \sin^2(\vartheta+\pi/2) = \cos^2\vartheta \end{align*}$$

The apparatus of the experiment has two PBS. Let us call the angles of the main axes of transmission $\zeta$ (zeta) for the left PBS and $\eta$ (eta) for the right PBS. For the orthogonal channels, the angles are $\zeta+\pi/2$ and $\eta+\pi/2$. Thus on the left side the retransmitted light pulse can be described either as $e_\zeta$ or $e_{\zeta+\pi/2}$, depending on which channel was taken. Similarly, on the right side, the retransmitted light pulse is $e_\eta$ or $e_{\eta+\pi/2}$.

Thus the facts of the experiment, taking into account *everything* stated so far, can be written as a point in a tensor space. In the following proposition, $P_1,P_2,P_3,P_4$ represent mutual probabilities of the possibilities represented by the matrix columns:

$$\begin{align*} \text{prop}_2 = &\space P_1\begin{pmatrix}e_\zeta&e_1\\e_\eta&e_2\end{pmatrix} + P_2\begin{pmatrix}e_{\zeta+\pi/2}&e_1\\e_\eta&e_2\end{pmatrix} \\ &+ P_2\begin{pmatrix}e_\zeta&e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} + P_2\begin{pmatrix}e_{\zeta+\pi/2} &e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} \end{align*}$$

Again the upper rows represent the left side of the apparatus and the lower rows represent the right side.

(A tongue-twisting exercise for the reader: Can you rephrase $\text{prop}_2$ in ordinary language?)

The probability of $\text{prop}_1 = {\begin{pmatrix}e_1&e_2\end{pmatrix}}^t$ is one, and thus each of $P_1,P_2,P_3,P_4$ equals the conditional probability of the left column, given the facts described by the right column (which in each case is $\text{prop}_1$). This is true by the definition of the conditional probability,

$$\begin{equation} \text{P}(\text{prop}_x|\text{prop}_y) = \frac{\text{P}(\text{prop}_x\wedge\text{prop}_y)}{ \text{P}(\text{prop}_y)} \end{equation}$$

where in this case $\text{P}(\text{prop}_y) = \text{P}(\text{prop}_1) = 1$ .

Although John S. Bell famously interpreted conditional probability to mean “no action at a distance,”[5] and used this interpretation to elide facts from the EPR-B experiment (whereas we made certain to include *every fact*), conditional probability *has no such meaning*. The *mathematical definition* of conditional probability is *exactly* what is written above. You can look this up in a text on probability theory. You can also verify, by playing with collections of ordinary objects, that conditional probability as defined above has *the following* intuitive meaning (rather than Bell’s): it is the probability $\text{prop}_x$ is true, given that $\text{prop}_y$ is true.

This means we calculate $P_1,P_2,P_3,P_4$ by assuming the light pulse has polarization $e_1$ on the left and $e_2$ on the right. In other words, we assume $\text{prop}_1$. We would have done so anyway, but it is nice to see the mathematics even subtly agreeing with us.[6]

Nothing in the formalism implies, as Bell assumes it does, anything about detections in opposite arms of the experiment affecting each other. Bell’s “encoding of no action at a distance” is meritless. Even Bell’s notation is not correct. The arguments of a probability should be propositions, not numeric quantities as Bell writes. Bell sloppily confuses probabilities with functions representing causal processes. We will not be so hasty. Let us proceed carefully and even use the $\stackrel{?}{=}$ symbol to indicate doubt.

A tentative formula for computing $P_1,P_2,P_3,P_4$ is as follows. If

$$\begin{equation} M_n = \begin{pmatrix} e_{\angle a} & e_1 \\ e_{\angle b} & e_2 \end{pmatrix} \end{equation}$$

then

$$\begin{equation} P_n \stackrel{?}{=} (\text{proj}_{e_{\angle a}}e_1)^2 (\text{proj}_{e_{\angle b}}e_2)^2 \end{equation}$$

which would be the mutual probability of the top row and bottom row, *if* there were no conditionality relationship between the rows. We do not have to do probability theory calculations to eliminate $e_1$ and $e_2$, because the projection operations eliminate these vectors. Furthermore, the probability of one polarizer setting *might* seem not to be conditional on the probability of the other polarizer setting, because both these settings are arbitrary. Thus $P_n$ *might* simply be the product of the probabilities of the two rows, as written above.

Or is it? We will see later on it is not so simple. In probability theory, where there *might* be an impediment, there likely *is* an impediment. What physicists such as Bell do not realize is that, once more and more facts become known, probabilities can be *inferred* “backwards in time.” Probability theory is about *inference*, not causation.

Now $\text{prop}_2$ can be written out more fully, but with doubt that what we are writing is correct:

$$\begin{align*} \text{prop}_2 = &\space P_1\begin{pmatrix}e_\zeta&e_1\\e_\eta&e_2\end{pmatrix} + P_2\begin{pmatrix}e_{\zeta+\pi/2}&e_1\\e_\eta&e_2\end{pmatrix} \\ &+ P_2\begin{pmatrix}e_\zeta&e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} + P_2\begin{pmatrix}e_{\zeta+\pi/2} &e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} \end{align*}$$

where

$$\begin{align*} P_1 &\stackrel{?}{=} \cos^2\zeta \, \sin^2\eta \\ P_2 &\stackrel{?}{=} \sin^2\zeta \, \sin^2\eta \\ P_3 &\stackrel{?}{=} \cos^2\zeta \, \cos^2\eta \\ P_4 &\stackrel{?}{=} \sin^2\zeta \, \cos^2\eta \end{align*}$$

The tensor $\text{prop}_2$ is a proposition stating the probability of each possible event in the two-channel Bell test experiment, in terms of angles $\zeta$ and $\eta$. Be careful, though, how you evaluate that proposition! Those angles are defined relative to $e_1$. But $e_1$ itself points in some unstated direction. How would you know whether the PBS were oriented correctly?

(This difficulty is resolved in the final subsection.)

We do, however, have a result that gives a (tentative) probability for every possible event in the experiment and which can be checked for summation to one:

$$\begin{align*} P_1 + P_2 + P_3 + P_4 &\stackrel{?}{=} \cos^2\zeta \, \sin^2\eta + \sin^2\zeta \, \sin^2\eta \\ &\mkern2em + \cos^2\zeta \, \cos^2\eta + \sin^2\zeta \, \cos^2\eta \\ &= (\cos^2\zeta) (\sin^2\eta + \cos^2\eta) \\ &\mkern2em + (\sin^2\zeta) (\sin^2\eta + \cos^2\eta) \\ &= \cos^2\zeta + \sin^2\zeta \\ &= 1 \end{align*}$$

# Correlation

## Expectation

Let us make a small diversion onto topics of probability theory. The first topic will be *expectation*.

In discrete probability theory (requiring no integration), an expectation is an average weighted by probabilities that add up to one. For example, suppose there are probabilities $P_i$ that add to one, and associated values $f(i)$. Then the expectation of $f$ is

$$\begin{equation} \text{E}(f) = \sum_i P_i f(i) \end{equation}$$

## Variance and standard deviation

Given the expectation of $f$, it would seem interesting to measure the difference between $f(i)$ and the expectation, but ignoring whether the difference is negative or positive. The sign can be ignored by squaring the difference.

Thus we have the *variance* of $f$:

$$\begin{align*} \text{Var}(f) = \sigma_f^2 &= \text{E}\left( (f - \text{E}(f))^2 \right) \\ &= \sum_i P_i \left( f(i) - \sum_{\mu} P_{\mu} f({\mu}) \right)^2 \end{align*}$$

The square root of the variance has the same dimensions as $f$ and is called the *standard deviation*:

$$\begin{equation} \sigma_f = \sqrt{\text{Var}(f)} = \sqrt{\sigma_f^2} \end{equation}$$

## Covariance and correlation

If there are two sets of values, $f(i)$ and $g(j)$, it can be useful to compute a value analogous to the variance, but involving both sets. Such a value is called the *covariance*:

$$\begin{align*} \text{cov}_{fg} = \sigma_{fg} &= \text{E}\left(\mkern1mu\left(f-\text{E}(f)\right) \left(g-\text{E}(g)\right)\mkern1mu\right) \\ &= \sum_{i,j} P_{ij} \left(f(i) - \sum_{\mu}P_{\mu} f({\mu})\right) \left(g(j) - \sum_{\nu} P_{\nu} g({\nu})\right) \end{align*}$$

A dimensionless value called the *correlation* is obtained by dividing the covariance of $f$ and $g$ by the respective standard deviations:

$$\begin{equation} \text{corr}_{fg} = \rho_{fg} = \frac{\sigma_{fg}}{\sigma_f\,\sigma_g} \end{equation}$$

The sign of a correlation depends on the definitions of $f$ and $g$ and tends to vary by author.

For the two-channel optical Bell test, there is a dimensionless figure physicists call the “quantum correlation” that equals $\pm\cos 2 \mkern1mu(\alpha-\beta)$, where $\alpha$ and $\beta$ are the PBS settings. If physicists are correct, this figure characterizes the preternatural “action at a distance” of the experiment, and should not be derivable without quantum mechanics. Only by recitation of the correct liturgy can the magic occur. No less than the Nobel prize was awarded for wizardry “proving” only quantum mechanics can give this result. Yet below we will derive $-\cos 2 \mkern1mu(\alpha-\beta)$ as the ordinary correlation, even though we use no quantum mechanics and assume only action by direct contact.[7]

# The correlation of the two-channel optical Bell test

## Solving two problems at once

Let

$$\begin{align*} M_1 &= \begin{pmatrix}e_\zeta&e_1\\e_\eta&e_2\end{pmatrix} & M_2 &= \begin{pmatrix}e_{\zeta+\pi/2}&e_1\\e_\eta&e_2\end{pmatrix} \\ M_3 &= \begin{pmatrix}e_\zeta&e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} & M_4 &= \begin{pmatrix}e_{\zeta+\pi/2}&e_1\\e_{\eta+\pi/2}&e_2\end{pmatrix} \end{align*}$$

Then

$$\begin{equation} \text{prop}_2 = P_1M_1 + P_2M_2 + P_3M_3 + P_4M_4 \end{equation}$$

Define $f$ as follows:

$$\begin{align*} f(M_1) &= f(M_3) &= +1 \\ f(M_2) &= f(M_4) &= -1 \end{align*}$$

Thus $f$ equals $+1$ if the left PBS retransmits in its main channel, $-1$ if it retransmits in its orthogonal channel.

The expectation of $f$ is tentatively [8]

$$\begin{align*} \text{E}(f) &\stackrel{?}{=} \cos^2\zeta \, \sin^2\eta - \sin^2\zeta \, \sin^2\eta \\ &\mkern2em +\cos^2\zeta \, \cos^2\eta - \sin^2\zeta \, \cos^2\eta \\ &= \cos^2\zeta - \sin^2\zeta \end{align*}$$

We would have liked for the expectation to equal zero. So let us use a trick.[9] Let us solve two equivalent problems simultaneously. That is, let us solve also for the case where the initial light pulses are $\begin{pmatrix} e_2&e_1\end{pmatrix}^t$. A weighted sum of the two initial light-pulse matrices gives $\text{prop}_3$, which has a symmetry $\text{prop}_1$ does not have:

$$\begin{equation} \text{prop}_3 = \frac12\begin{pmatrix}e_1\\e_2\end{pmatrix} + \frac12\begin{pmatrix}e_2\\e_1\end{pmatrix} \end{equation}$$

Remember that in $\text{prop}_1$ the axis of transmission is not specified. Thus $\text{prop}_3$ is, for practical purposes, a replacement problem for $\text{prop}_1$. The new proposition says, “The orthogonal planes of polarization have one half probability of being oriented a certain unspecified way, and one half probability of being oriented orthogonally to that. Which light pulse goes which way is even chance.” The probability of given axes and light pulse directions is the same in $\text{prop}_1$ and $\text{prop}_3$. The difference is in the algebra, which quickly becomes unmanageable without the trick.[10]

We now need four more matrices:

$$\begin{align*} M_5 &= \begin{pmatrix}e_\zeta&e_2\\e_\eta&e_1\end{pmatrix} & M_6 &= \begin{pmatrix}e_{\zeta+\pi/2}&e_2\\e_\eta&e_1\end{pmatrix} \\ M_7 &= \begin{pmatrix}e_\zeta&e_2\\e_{\eta+\pi/2}&e_1\end{pmatrix} & M_8 &= \begin{pmatrix}e_{\zeta+\pi/2}&e_2\\e_{\eta+\pi/2}&e_1\end{pmatrix} \end{align*}$$

And I will leave it as an exercise for the reader to show that the new (but still questionable) probability formula is

$$\begin{equation} P_n' \stackrel{?}{=} \begin{cases} \frac12(\text{proj}_{e_{\angle a}}e_1)^2 (\text{proj}_{e_{\angle b}} e_2)^2 & \text{if } n\in\{1,2,3,4\} \\ \frac12(\text{proj}_{e_{\angle a}}e_2)^2 (\text{proj}_{e_{\angle b}}e_1)^2 & \text{if } n\in\{5,6,7,8\} \end{cases} \end{equation}$$

so that

$$\begin{align*} P_1' &= P_8' \stackrel{?}{=} (1/2)\cos^2\zeta \, \sin^2\eta \\ P_2' &= P_7' \stackrel{?}{=} (1/2)\sin^2\zeta \, \sin^2\eta \\ P_3' &= P_6' \stackrel{?}{=} (1/2)\cos^2\zeta \, \cos^2\eta \\ P_4' &= P_5' \stackrel{?}{=} (1/2)\sin^2\zeta \, \cos^2\eta \end{align*}$$

Define

$$\begin{align*} f(M_5) &= f(M_7) &= +1 \\ f(M_6) &= f(M_8) &= -1 \end{align*}$$

so $f$ still means $+1$ for the main channel. Then

$$\begin{align*} 2\,\text{E}(f) &\stackrel{?}{=} \cos^2\zeta \, \sin^2\eta - \sin^2\zeta \, \sin^2\eta \\ &\mkern2em +\cos^2\zeta \, \cos^2\eta - \sin^2\zeta \, \cos^2\eta \\ &\mkern3em +\sin^2\zeta \, \cos^2\eta -\cos^2\zeta \, \cos^2\eta \\ &\mkern4em + \sin^2\zeta \, \sin^2\eta -\cos^2\zeta \, \sin^2\eta = 0 \end{align*}$$

as desired.

A desirable value for the standard deviation, is one. Let us see what we get:

$$\begin{align*} 2\,\sigma_f^2 &\stackrel{?}{=} (\cos^2\zeta \, \sin^2\eta)\,(+1 - 0)^2 \\ &\mkern1.5em + (\sin^2\zeta \, \sin^2\eta)\,(-1 - 0)^2\\ &\mkern2.0em +(\cos^2\zeta \, \cos^2 \eta)\,(+1 - 0)^2 \\ &\mkern2.5em + (\sin^2\zeta \, \cos^2\eta)\,(-1 - 0)^2\\ &\mkern3.0em +(\sin^2\zeta \, \cos^2 \eta)\,(+1 - 0)^2 \\ &\mkern3.5em +(\cos^2\zeta \, \cos^2\eta)\,(-1 - 0)^2\\ &\mkern4.0em +(\sin^2\zeta \, \sin^2 \eta)\,(+1 - 0)^2 \\ &\mkern4.5em +(\cos^2\zeta \, \sin^2\eta)\,(-1 - 0)^2 = 2 \end{align*}$$

Indeed, then, $\sigma_f \stackrel{?}{=} 1$ as desired.

Define a function $g$ that is similar to $f$, but responds to the right-hand PBS instead of the left:

$$\begin{align*} f(M_1) &= f(M_3) = f(M_5) = f(M_7) &= +1 \\ f(M_2) &= f(M_4) = f(M_6) = f(M_8) &= -1 \\ g(M_1) &= g(M_2) = g(M_5) = g(M_6) &= +1 \\ g(M_3) &= g(M_4) = g(M_7) = g(M_8) &= -1 \end{align*}$$

The function $g$, like $f$, has (tentative) expectation zero and (tentative) standard deviation one.

On either side, which channel has positive sign and which negative is a matter of preference. With some definitions of $f$

and $g$ the correlation will have a plus sign in front of a “$\cos$” and with other definitions a negative sign, and again it makes no real difference. Sometimes you will see it one way, sometimes the other. Our result will have a minus sign.

## A naïve attempt to derive the correlation of $f$ and $g$

Because the standard deviations of $f$ and $g$ are both one, the covariance and correlation of $f$ and $g$ are equal to each other. In general they would be different at least in that correlation is dimensionless and covariance is not, but our $f$ and $g$ already are dimensionless, so our covariance is dimensionless.

We can thus compute the covariance and treat the result as the correlation.

Let us try to derive the covariance of $f$ and $g$ by the formula

$$\begin{equation} \text{cov}_{fg} \stackrel{?}{=} \mkern-10mu \sum_{i,j\in \{1,\ldots,8\}} \mkern-10mu P_i P_j\,f(M_i) g(M_j) \end{equation}$$

This results in expressions similar to those of the CHSH inequality.

A little thought reveals a tremendous problem with this approach: the mathematics does not distinguish effects that are due to $\zeta$ from those that are due to $\eta$. Polarizers could be stuck willy-nilly within the apparatus, but the mathematics could not tell the difference. *We have made unwarranted assumptions about the mutual probabilities* $P_{ij}$. We have failed to consider the $P_{ij}$ might not be products of $P_i$ and $P_j$. The resulting expressions are useless.

The CHSH inequality also is useless.

That physicists are *permitted* to be so immensely incompetent in probability theory is scandalous. They teach equal incompetence to students and a next generation, who are fleeced of their possessions for the privilege of being made so stupid. Global peace and the defense of the United States are threatened. No scientific field should be allowed such ignorance.

My education was in signal processing, in the 1980s before electrical engineering was so badly corrupted with “quantum” nonsense. I look for the *actual* solution to a problem such as the Bell test, rather than for some fantasy about entangled fairies and teleporting unicorns. So I work at it until an effective method comes to mind.

What works is to *specialize* the problem so there is only one polarizer setting. Then that specialized solution can be generalized. (This is the reappearance of *specialization*, forecasted in [9])

## A solution for $\eta=0$

If $\eta=0$, the symbol $\eta$ disappears from expressions, and indeed we need no PBS on the right. A channel is either totally open or totally closed.

The difficulty with expressions containing both $\zeta$ and $\eta$ exists no longer. Effectively there is no right-side PBS. Now also all effects on the left are from the left-side PBS and are expressed through $\zeta$.

We now have

$$\begin{align*} P_1' &= P_8' = 0 \\ P_2' &= P_7' = 0 \\ P_3' &= P_6' = (1/2)\cos^2\zeta \\ P_4' &= P_5' = (1/2)\sin^2\zeta \end{align*}$$

and can compute the correlation correctly:

$$\begin{align*} \rho_{fg} = \text{cov}_{fg} &= (1/2)\,(\cos^2\zeta)\,(f(M_3)g(M_3) + f(M_6)g(M_6)) \\ &\mkern2em + (1/2)\,(\sin^2\zeta)\,(f(M_4)g(M_4) + f(M_5)g(M_5)) \\ &= -(\cos^2\zeta - \sin^2\zeta) \\ &= -\cos2\mkern1.2mu\zeta \\ &= -$$

$$\cos2\mkern1mu(\zeta - \eta)$$

The next to last step is by one of the double-angle relations earlier quoted from the *CRC Handbook of Mathematical Sciences*. The last step is true because $\eta=0$.

The result is the same as the “quantum correlation” for the given PBS settings. But there is still one more step before we get the complete answer.

## Realizing that the plane of polarization does not matter

That step is to add zero to $\zeta - \eta$.

But not just any zero. Let $\delta$ be any angle whatsoever. Also let $\alpha=\zeta+\delta$ and $\beta=\eta+\delta$. Then

$$\begin{align*} -\cos 2 \mkern1mu(\alpha-\beta) &= -\cos 2 \mkern1mu( ( \zeta+\delta ) - ( \eta+\delta ) ) \\ &= -\cos 2 \mkern1mu( ( \zeta-\eta ) + ( \delta-\delta ) ) \\ &= -\cos 2 \mkern1mu( ( \zeta-\eta ) + 0 ) \\ &= -\cos 2 \mkern1mu( \zeta-\eta ) \end{align*}$$

Thus the PBS settings can be rotated together to any new calibration, without this rotation changing the form of the correlation. However, let us assume there is a standard calibration for a PBS. We cannot recalibrate. There remains, however, an equivalent interpretation: that the polarization angles of the initial light pulses do not matter towards the correlation, as long as the two planes of polarization are orthogonal to each other.

*This is because the correlation has turned out to be a function of the* relative *angle of transmission axes, expressed as a cosine.* The cosine function is equivalent to a dot product, which is a measure of the angle between two vectors.

We needed this invariance over in-unison rotation of the PBS settings, anyway. We had no way to know how to set the PBS, otherwise. *The PBS axes must be set relative only to each other, not relative to the light source.*

There is, by the way, a factor of two in $-\cos 2 \mkern1mu(\alpha-\beta)$ that *does not* appear if the experiment were done with spin-1 particles. Mathematically, the only difference is a linear transformation of the independent variables. Physically, I would give polarizing filters the “projection of magnitude” interpretation, whereas for spin-1 particles the device (such as a Stern-Gerlach magnet) is perhaps more like a rectangular window that is open only a certain amount. I base this interpretation on some investigation of trigonometric identities.

In any instance, the goal is achieved. I have derived the correlation of the two-channel optical Bell test, without quantum mechanics and assuming only action by direct contact. The correlation is

$$\begin{equation} \rho_{fg} = -\cos 2 \mkern1mu(\alpha-\beta) \end{equation}$$

where $\alpha$ and $\beta$ are the respective axes of transmission of the main channels of the two PBS.[11]

This is the same expression as the so-called “quantum correlation”—and it is correct, even if I made a derivation error somewhere—but we can see that “quantum” is a misnomer. It is the ordinary correlation from probability theory. It also can be derived from classical coherence theory of electromagnetic waves.[12] The only thing mysteriously “quantum” about the expression is how any “quantum” physicist obtained it by *some* means, given the field’s habit of uncritically publishing *mistakes* such as Bell’s and Clauser’s.

Indeed, given the “quantum correlation,” and recognizing it is invariant to in-unison rotations of $\alpha$ and $\beta$, then, setting $\beta=0$, one could have worked backwards and shown that no quantum mechanics was necessary for a derivation. That no such “reverse derivation” appears in the “elite journals” is testimony to the incompetence of a physics community that awarded the Nobel prize to Clauser, Aspect, and Zeilinger. The research of these Nobel laureates should have been *withdrawn* rather than awarded.

Bell's *Speakable and Unspeakable in Quantum Mechanics* [13] ought to be reclassified as "Pathological Science." It would be one of the most pathological examples.

Let me finish by repeating my harshness is because the defense of the United States is at stake, and because students are bankrupted to be taught nonsense. "Collegial politeness" is unaffordable. In any case I am not a colleague of "professional physicists," as they have sometimes reminded me. I am an amateur with no institutional affiliation, my education is in electrical engineering, and my trade was computer programming. I am expected to be polite to professional physicists, but the reverse is not expected. Gone are the days when Benjamin Franklin or Michael Faraday could become a Fellow of the Royal Society.

An excuse will be made that "specialization is necessary now and non-specialists cannot fathom the details," and yet here they are, physicists, pretending to be able to do random process analysis without even bothering to study the subject. A reasonable conclusion is they believe they are so smart they need not study the subject (which I as a signal processing student had to, so I could study random process subjects such as "sphere hardening" and then have devices I might design actually *function*). There is no need for physicists to seek assistance from disciplines they themselves have not mastered. They can intuit it all with their gigantic brains, so much bigger than mine and yours. And they believe Benjamin Franklin and Michael Faraday were too stupid to do what today's so very "professional" and specialized physicists do.

If today's physicists really were so smart compared to Franklin and Faraday, they would not believe in entangled fairies and teleporting unicorns. And it is not true that specialization is necessary now. Physicists have demonstrated, indeed, that a tightly knit community of specialists causes isolation and orthodoxy. They are a community divorced from mathematics, logic, and epistemology.

And so I finish in *this* subsection, rather than with a separate formal conclusion, because failure to realize one is free to recalibrate the apparatus is perhaps the worst failure to read the plain meaning of a mathematical expression any scientists have ever made. It is a pratfall, and the Nobel prize was awarded for this pratfall. $\square$ [14]

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1. Including *Science*, to which I subscribe.
2. The $\varepsilon \text{-}\delta$ limits of a field represent activity by direct contact. The "infinitesimals" of nonstandard analysis also must do so, although in a way less like a mechanical iris closing in. Being an engineering graduate, I prefer the intuition of a mechanical iris.
3. W. H. Beyer, ed., *CRC Handbook of Mathematical Sciences*, 6th ed., CRC Press, Boca Raton, FL, 1987.
4. Or of "photons," if I had wanted to use a designation with stronger connotations than "light pulses." I do not want to.
5. J. S. Bell, *Bertlmann's socks and the nature of reality*, Preprint CERN-TH-2926, CERN, Geneva, 1980. https://cds.cern.ch/record/142461/files/198009299.pdf
6. Incredibly, Bell's "encoding of no action at a distance" alters $\text{prop}_1$, even though we know it to be true.
7. I believe you will also find, if you go to the trouble, that violations of Bell or CHSH inequalities are inconsistent with this correlation function, no matter how it be derived. The inequalities are incorrect mathematics. The experimental technique of Alain Aspect and others must be flawed, for it should be impossible to obtain violations of the inequalities.
8. I suspect this is a correct answer, despite that it was derived by questionable means. My reason is that $\eta$ disappears from the answer.
9. There is no "physics" in this trick. It is a *mathematical* maneuver to take advantage of symmetry, and I would classify it as an example of what George Pólya called the *specialization* heuristic. (G. Pólya, *How to Solve It: A New Aspect of Mathematical Method*, expanded Princeton Science Library ed., Princeton University Press, Princeton, NJ, 2004, With a foreword by John H. Conway.) The specialization heuristic will reappear later.
10. I wonder if without the trick we would have had to use calculus.
11. Some joker may come along and insist I assumed "instantaneous action at a distance" at such and such a point. Please feel free to laugh at them. They are making a fool of themself.
12. A. F. Kracklauer, *EPR-B correlations: non-locality or geometry?* J. Nonlinear Math. Phys. 11 (2004), suppl., 104–109. MR2117173. https://doi.org/10.2991/jnmp.2004.11.s1.13
13. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge Univ. Press, Cambridge, 1987.
14. 